International Journal of Solids and Structures 41 (2004) 619–635
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
www.elsevier.com/locate/ijsolstr

Multi-objective and multi-loading optimization of ultralightweight truss materials

Jing-Sheng Liu [a], Tian Jian Lu [b,*]

[a] *Department of Engineering, University of Hull, Hull HU6 7RX, UK*
[b] *Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, UK*

Received 24 September 2003; received in revised form 24 September 2003

Abstract

A multi-parameter optimization procedure for ultralightweight truss-core sandwich panels is presented in this paper. To satisfy high design requirements, the proposed approach incorporates objectives from various structural performances of the material system at many loading cases simultaneously. To determine an optimal topology of the truss-core panel, the procedure combines standard finite element (FE) analysis with structural system profile analysis and multi-factor optimization techniques. Detailed configuration and sizing for both facesheets and individual struts in the sandwich panel are optimized. Two examples are presented: (i) tetragonal core panel, and (ii) plagihedral pyramidal core panel. The optimization improves the structural performance of each panel under multiple loading cases and minimizes its structural mass simultaneously.

Keywords: Multi-objective optimization; Ultralightweight lattice materials; Sandwich panels; Structural optimization; Finite element method

1. Introduction

Ultralightweight metallic lattice materials have recently been made with novel manufacturing techniques. They are basically sandwich systems consisting of a 3-D network of fully triangulated struts or plates. Fig. 1 taken from Evans et al. (2001) shows some typical sandwich panels with a truss core and two thin facesheets, and a sandwich panel with honeycomb core for comparison (Evans et al., 2001). The facesheets take the membrane and bending loads while the core resists the shear loads and compression at the loading points. Potential applications include replacements for honeycombs (Evans et al., 2001), low resistance filtration systems, and multi-functional materials (e.g., heat dissipation in addition to load-carrying capacity (Kim et al., in press; Tian et al., submitted for publication; Sypeck and Wadley, 2001; Wicks and Hutchinson, 2001; Wallach and Gibson, 2001)). Compared with other flat panels in bending

* Corresponding author. Tel.: +44-122-3766316; fax: +44-122-3332662.
E-mail address: tjl21@cam.ac.uk (T.J. Lu).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2003.10.003

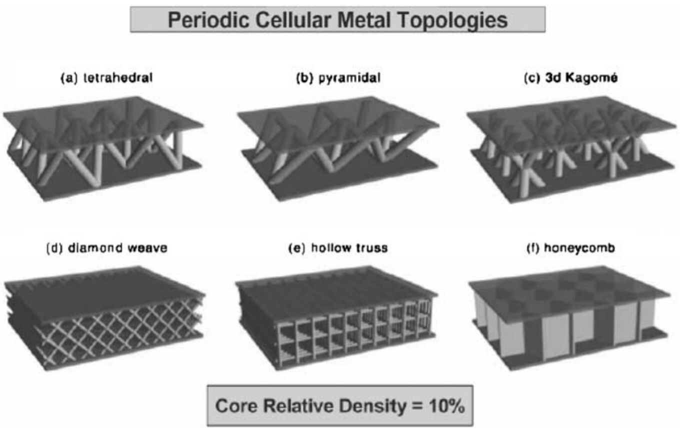

Fig. 1. Topological comparison of periodic structures.

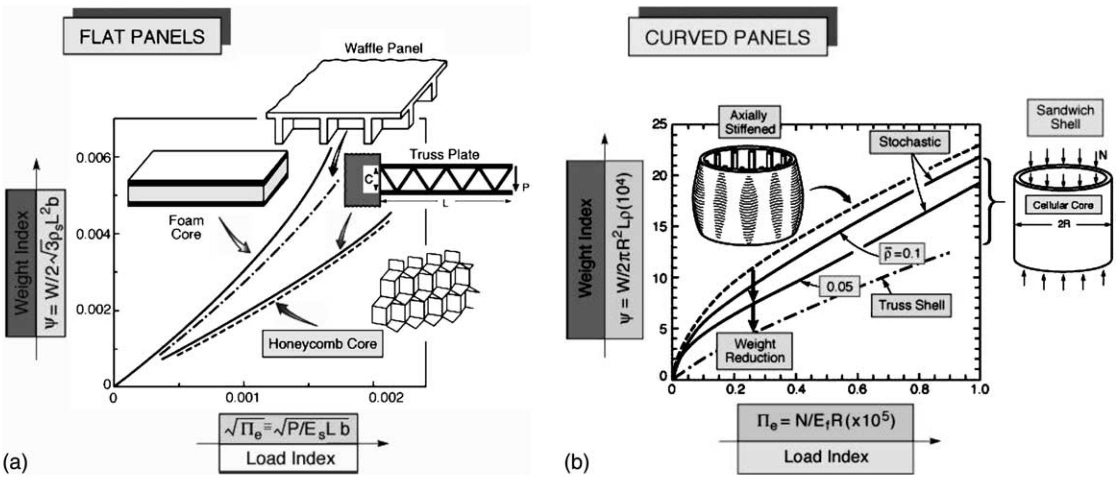

Fig. 2. Comparison of minimum structural weight of competing sandwich panels: (a) flat panels subjected to bending, and (b) curved panels under axial compression (Evans et al., 2001).

(Fig. 2a, taken from Evans et al., 2001), it has been established (Evans et al., 2001; Deshpande et al., 2001; Deshpande and Fleck, 2001) that panels having cores with a stochastic cellular topology (e.g., metal foams) are not as weight efficient as truss core panels with periodic cellular topologies. For curved panels subjected to compression (Fig. 2b, taken from Evans et al., 2001), the benefits of using configured cores increase as a result of the biaxial nature of the strain.

The minimum weight design of sandwich plates comprised of tetrahedral truss core and either planar trusses or solid face sheets subjected to combined bending and transverse shear loads is apparently first studied by Wicks and Hutchinson (2001). The optimization is subjected to the constraints of face member yielding and buckling as well as core member yielding and buckling, and a sequential quadratic programming algorithm is used to solve the optimization problem. Compared with sandwiches with hexagonal honeycomb cores that are widely considered as the lightest weight plate structures, the optimized sandwich plates with truss cores are found to be equally effective, but the latter have the advantage of having an open core that allows for cooling, actuation and other functions (Wicks and Hutchinson, 2001). Wallach and Gibson (2001) used the unit cell approach to calculate the elastic moduli, uniaxial compressive strength and shear strength of a similar structure with a pyramidal truss core as functions of the relative density and aspect ratio of the structure; the predictions agree well with experimental measurements. Deshpande and Fleck (2001) studied the minimum weight of sandwich beams with a tetrahedral truss core and solid facesheets subjected to 3-point bending. The strength formulae (face-yield, face-wrinkling and core shear) for the sandwich are used and a graphical method based on collapse mechanism map is used for optimization. It is found that optimal sandwich designs lie at the boundaries of competing collapse modes. However, the strut aspect ratio is kept constant during optimization, and hence the resulting minimum weight design is about 30% greater than the global minimum obtained by Wicks and Hutchinson (2001).

Previous optimization studies on the structurally efficient truss core panels (Wicks and Hutchinson, 2001; Wallach and Gibson, 2001; Deshpande and Fleck, 2001) consider mostly a single objective function (weight) and simple loading cases (bending and/or transverse shear). The possibility of achieving a design that efficiently optimizes multiple performances under multiple loading cases, coupled with the difficulty in selecting the values of a large set of design variables, makes structural optimization an important tool for the design of lightweight systems. The focus of this work is to seek detailed optimal design of truss-core sandwich panels: in the optimization, structural strength, stiffness, natural frequencies, and structural mass were optimized simultaneously, while considering many loading cases that may affect the performance of the system.

A multi-objective optimization computer program, MOST (Multifactor Optimization of Structures Technique), has been developed to accommodate and implement the optimization method. The MOST system utilizes commercially available FE codes (e.g. ABAQUS or ANSYS) as its analysis program, and its optimization system control program is written in UNIX shell scripts. It can control optimization flow and execute application programs in UNIX environment so that all functions can be integrated in a system. The program can account for various environment conditions and loading scenarios. The optimization method treats complex engineering design problems, which may have multiple objectives and multiple loading cases, in a systematic way by employing a parameter profile analysis. The method introduces an assessment system which brings scores and merit indices into the range of 0–10 for all non-commensurable performances (with different units) and loading cases. The formulation makes no difference between performance constraints and objective functions. These features make MOST a powerful, cost-effective, and reliable tool to optimize complex structural systems. The method has been successfully used to optimize large reflector antenna systems and fiber-reinforced composite panel structures (Liu and Hollaway, 1998, 2000).

The optimization of a truss core panel demonstrates further the capabilities of the optimization methodology and program system. The optimization results show that the optimization procedure can greatly improve the material performances at almost all the loading cases considered.

2. Cellular topology of truss core and loading cases

We consider a sandwich panel clamped at one end and free at the other (Fig. 3a or c), with solid facesheets and a truss core with either tetrahedral cells (Fig. 3b) or plagihedral pyramidal cells (Fig. 3d). Within each panel, the tetrahedral (or plagihedral pyramidal) cells may be identical or not. Other periodic topologies exist for the core (Fig. 1), but for the purpose of illustrating the multi-parameter optimization technique, the focus of this study is placed on truss structures with tetrahedral and plagihedral pyramidal cells.

Fig. 4 shows a truss-core panel with tetrahedral cells made from a casting aluminum alloy (LM25) with composition Al–Si7–Mg0.3 (wt%). Fig. 4a presents the topology of the panel viewed from the x_1-axis, whereas Fig. 4b is the image view from the x_2-axis. To manufacture the panel, the tetrahedral core was first

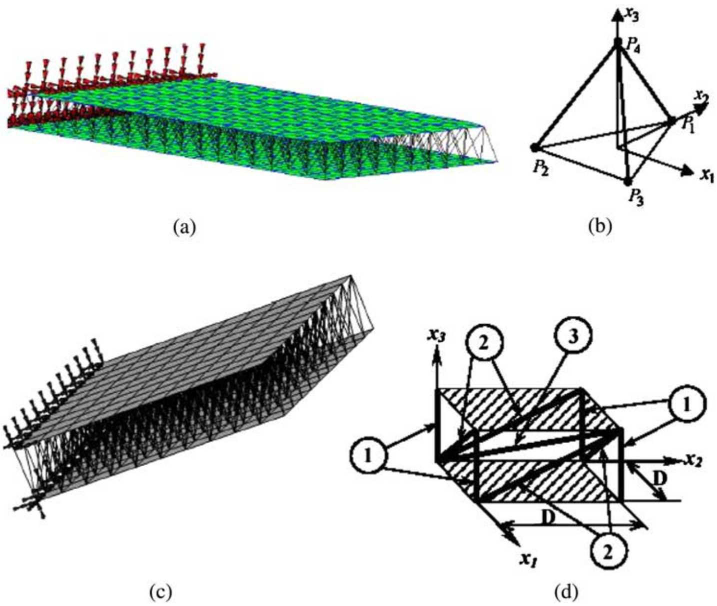

Fig. 3. Truss-core sandwich panel clamped at one end and free at the other: (a) tetragonal core topology of preliminary design, (b) tetrahedral unit cell, (c) plagihedral pyramidal core topology of preliminary design, and (d) plagihedral pyramidal unit cell.

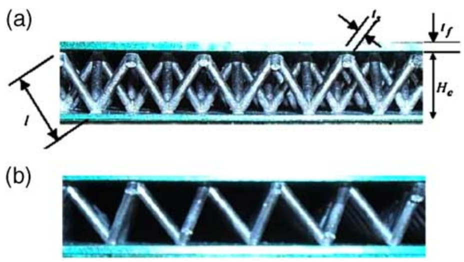

Fig. 4. Sandwich panel with tetrahedral truss core: (a) viewed from x_1-direction, and (b) viewed from x_2-direction (cf. Fig. 3b).

injection molded in polystyrene. This polystyrene lattice was then used as the sacrificed pattern in a "lost-wax" investment casting to produce the LM25 panel (Deshpande et al., 2001; Deshpande and Fleck, 2001). A similar manufacturing technique (the Jamcorp process) was used to fabricate the truss core panels tested by Wallach and Gibson (2001).

Chiras et al. (2002) experimentally and then, by using the method of finite elements, Hyun et al. (2002) found that, the tetragonal core is anisotropic in shear with two soft orientations, both governed by the onset of plastic buckling. Optimization in one orientation is therefore not applicable to other orientations. To ensure that the truss-core sandwich panel is to be effective in various applications, a range of loading cases will be selected to study the performance of each panel. Numerical results on maximum displacement, maximum stress in the struts and maximum von Mises stress in the facesheets, fundamental frequency, and total mass will be presented for selected loading conditions, obtained from static analysis and normal modes analysis. The details of the microstructures and performance results for both the initial and optimized panel designs will be given.

3. Design evaluation principles

The requirements for a material design often dictate that the optimization involves quantity-variant objectives, constraints, loading cases and design variables. An important aspect of the optimization process is the establishment of suitable evaluation criteria. The complex cross-relationships raise the question of how to appraise the design in order to yield an overall, quantitative performance index which brings out the character of the system. A systematic method for evaluating a design is presented below. Based on the concept of parameter profiles (Thompson and Goeminne, 1993; Liu and Thompson, 1996), this method reviews, in a non-dimensional manner, the performance profile of the material system by simultaneously considering many individual performance parameters at a spectrum of loading cases, in addition to considering mass and cost.

3.1. Performance data matrix and parameter profile matrix

The basis of the analysis method is a $m \times n$ matrix of data which describes system performance under different loading cases. The matrix, called performance data matrix (PDM), is a schematic representation of a collection of data. The matrix lists every item of the loading cases considered as well as every performance parameter relevant to each individual loading case. The matrix is defined by the set of performance parameters P_i ($i = 1, \dots, m$) included in the analysis at loading cases C_j ($j = 1, \dots, n$). Thus, data point d_{ij} of the matrix represents the performance of the system with respect to performance P_i at case C_j.

Each data point is obtained by performing conventional structural analysis. However, the actual values of the analytical results are only of limited use. The character of the system may be adequately assessed by a review of the profile of the performances at various loading cases with respect to the proximity of actual performances to their acceptable limits and the best level values of the performances.

An evaluation matrix called parameter profile matrix (PPM) is subsequently created in the assessment method. The data point D_{ij} in the PPM is a non-dimensional number in the range 0–10, which is determined by the closeness of the actual performance d_{ij} to the acceptable limit and the best level value of the performance.

The calculation of the data point D_{ij} for only one acceptable limit (e.g. lower limit) is, in principle, as follows:

$$D_{ij} = \frac{d_{ij} - l_{ij}}{b_{ij} - l_{ij}} \times 10 \tag{1}$$

where d_{ij} is the actual value of the performance taking from the PDM; l_{ij} and b_{ij} are the lower limit and best level value, respectively. Eq. (1) is valid for $l_{ij} < d_{ij} < b_{ij}$; for $d_{ij} > b_{ij}$, $D_{ij} = 10$, and for $d_{ij} < l_{ij}$, $D_{ij} = 0$. The data point D_{ij} for the cases of acceptable upper limit and double acceptable limits can be calculated in a similar way.

3.2. Performance assessment

The information obtained from the parameter profile matrix makes it possible to evaluate the system as a whole. For each row and column of the PPM matrix, the mean and standard deviations (SDs) for each parameter and loading case are calculated. The SD is a measure of the degree of dispersion of the data around the mean. A well-designed system should have a low SD and a high mean close to 10. The existence of high SDs signifies that the system is likely to have significant problematic areas. Therefore, a high SD for a row indicates a variable system performance at different loading cases for a particular parameter. On the other hand, a high SD for a column indicates that the system, at the specified loading case, is likely to have significant problematic performance.

It is possible to analyze the system performance at a more advanced level. A parameter performance index (PPI) and a case performance index (CPI) can be defined, as follows:

$$(\mathrm{PPI})_i = \frac{n}{\sum_{j=1}^{n} 1/D_{ij}} \quad (i = 1, \ldots, m), \quad \text{and} \quad (\mathrm{CPI})_j = \frac{m}{\sum_{j=1}^{m} 1/D_{ij}} \quad (j = 1, \ldots, n) \tag{2}$$

When the ith parameter is very vulnerable, some data points D_{ij} of the PPM will have values close to 0 and hence the $(\mathrm{PPI})_i$ will also be close to 0. Similarly, when the system is vulnerable at the jth loading case, $(\mathrm{CPI})_j$ will be close to 0.

The mean values, SDs, PPIs and CPIs, provide separately an overall performance rating for each system and each loading case. The indices are calculated by summing the inverse of the data points to avoid the effect of any particularly low scores being hidden by high scores in other respects, and this is possible when only the mean is calculated. For convenience, the values of performance indices are limited to the range 0–10, no matter how many data points are used in each calculation. This enables different parameters and loading cases to be compared in order to gain an overall perspective of the character of the system. The system can be evaluated by using this information, as below:

(a) A comparison of PPIs will indicate whether the system performs better with respect to some performances than others.
(b) A comparison of CPIs will show whether the system performs significantly better at some loading cases than others.

The weakest loading case can be identified by comparing the values of CPIs. Once this is specified, it is possible to determine which performance parameter has the most influence on that weak behavior. If this is given for one item only, then it is sufficient to search the particular column for the lowest performance data point. This will identify the parameter for which the system performance needs to be improved.

4. The optimization problem

The performance data matrix embodies a variety of performance parameters of a complicated system at various loading conditions. To optimize such a system signifies to improve these performance parameters under these loading cases as well as the side constraints of the design variables.

The above non-linear programming problem has a large number of objectives, variables and constraints. In real problems these objectives are usually antagonist functions. In mathematical terms, to optimize some of the functions may degrade others. For this reason and owing to the complexity of the material system, it may not be possible to find a simple closed-form relationship that includes all functions and design variables.

In this work, a goal system is established by transforming every performance parameter into a set of goal functions in a range of 0–10 with respect to the loading cases. For every (non-dimensional) performance parameter, the target has the same specified value of 10. The value of a goal function reflects how close it is to the predetermined target. Consequently, the original optimization problem is converted to a problem of minimizing the deviations between all goal functions and their pseudo-targets. This is equivalent to minimizing the distance between the performance P_i and its given best value P_i^*:

$$\min \|P_i - P_i^*\|, \quad i = 1, \ldots, m \tag{3}$$

It is known from matrix profile analysis that the PPI is a measure of the vulnerability of each performance parameter and the CPI is a measure of the vulnerability of each loading case. To obtain a measure of the vulnerability of a particular parameter/loading-case combination, the PPI and CPI, therefore, should be integrated. A high vulnerability results in low indices and a high superiority results in high indices. A single multiplication therefore appears most appropriate:

$$S_{ij} = \mathrm{PPI}_i \times \mathrm{CPI}_j, \quad i = 1,2,\ldots,m; \; j = 1,2,\ldots,n \tag{4}$$

Each performance parameter or loading case is weighted according to its importance, and the data points are calculated as

$$S_{ij} = Wp_i \cdot \mathrm{PPI}_i \times Wc_j \cdot \mathrm{CPI}_j, \quad i = 1,2,\ldots,m; \; j = 1,2,\ldots,n \tag{5}$$

where the Wp_i and Wc_j are weighting factors in the range of 0–1 reflecting the preference for different parameters and different loading cases.

The above design synthesis concept provides a framework for formulating the quantifiable portion of a system design, from which advanced optimization techniques can be developed. The optimization objective function should be selected to measure the overall design quality of a structural system. An overall performance index (OPI), which is a qualitative score, can be established for the system considering all the performances and loading cases. Mathematically, this can be expressed as (for the weighted case)

$$\mathrm{OPI} = \frac{100}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} S_{ij} \tag{6}$$

The function OPI has a value in the range of 0–100, comprising all the m parameters considered under a total of n different loading conditions. The use of optimization techniques makes it possible to force some or all performances to approach their best level values. The nearer the performances are to their acceptable limits, the more severe will be 'the punishments' (penalties).

The optimization problem as stated above is complicated by the fact that, for some engineering problems, the objective function does not always have continuous first and second derivatives. Consequently, the problem has often been tackled by means of numerical processes. However, the numerical calculation of the gradient and Hessian matrix may be costly, if not impossible. Therefore, in the present study, the optimization problem is solved by maximizing the objective function by using the effective zero-order method which utilizes conjugate search directions. An effective polynomial interpolation uni-dimensional search technique is also used in the algorithm. The procedure consists of analyzing a preliminary starting design, sensitivity analysis, and the development of preferential values of the design variables. The analysis step may be mathematically 'exact', but non-linearities in the system response with respect to changes in the design variables make the development step approximate. These two steps are repeated iteratively to achieve the final design. The utilization of the above optimization methodology will be demonstrated below for ultralightweight truss-core panels.

5. The optimization of truss-core panels

Optimizing material properties and minimizing weights and/or costs is the most critical aspect in designing a metallic truss-core system. It is of practical interest to design a truss-core panel of high strength and stiffness at minimum weight. Thus, the deflection from the unloaded configuration forms one of the objective functions to be minimized in the design optimization. As truss-core panels are used as a structural material, they should behave such that stresses distribute as uniform as possible under a variety of loading scenarios. Therefore the stress levels in constitutive members (struts and facesheets) need to be minimized. Since lightweight structures are desirable from the point of view of reducing manufacturing as well as operating costs, mass is taken as one of the objectives to be minimized. The configuration of the material must be stiff enough to avoid undesirable interaction with any mechanical vibration and to withstand various external forces without suffering from excessive distortions. Hence structural fundamental frequencies are also included for optimization.

In summary, the optimization is required to minimize maximum displacement and maximum stresses in local members, to increase fundamental frequency to the prespecified (target) level, and to minimize structural mass. All of these topics are of great importance to the design of truss-core panels.

5.1. Optimization of a truss-core panel with identical tetrahedral cells under three loading cases

The sandwich panel has a length of L, width of B, core height of H_c, and face thickness t_f. Each tetrahedral cell consists of identical cylindrical struts of cross-sectional area A (diameter $t_s = (4A/\pi)^{1/2}$) and length l, and its nodes are denoted P_1–P_4 as shown in Fig. 3b. The angle between the truss structs and the faces is $\varphi_c = \sin^{-1}(H_c/l)$. The base plane of the tetrahedral cell $P_1P_2P_3$ is parallel to the x_1x_2 plane (i.e., parallel to the facesheets), and the x_1-axis is parallel to the line connecting nodes P_2 and P_3. The $x_1x_2x_3$ coordinate system corresponds to the threefold symmetry of the truss material about the x_3-axis. For preliminary design, the following values are selected: $t_f = 1$ mm, $A = 0.25$ mm^2, $l = 11.55$ mm, $L = 100$ mm, $B = 100$ mm, and $H_c = 10$ mm. The truss core and the solid facesheets are made of the same steel material with Young's modulus $E_s = 206$ GPa, Poisson's ratio $\nu = 0.3$, and mass density $\rho = 7850$ kg/m^3. The total mass of the panel before optimization is $W = 0.1643$ kg. Other performances of the initial design can be found in Table 1. As the main objective of this study is to develop a multi-factor optimization methodology and demonstrate its applicability through truss-core materials, for simplicity, the metal material will be restricted to deform within the elastic range for all loading cases. However, incorporating various failure criteria of yielding and elastic/plastic buckling into the optimization procedure is achievable (Wicks and Hutchinson, 2001; Lu et al., 2001; Lu and Evans, 2002).

The optimization study aims to determine possible new configuration and sizing as candidate for a new system. The design variables used for the optimization of this one-side clamped truss-core panel are

- facesheet thickness,
- cross-sectional area of all struts,
- panel height.

These sizing and geometric variables will be considered simultaneously in the optimization. Generally speaking, the stiffness and strength of a sandwich panel will be improved if its facesheets are moved further apart (increasing the core height). However, as a cost, this will increase the weight because all the struts in the core will become longer. Since mass and stiffness/strength are all included as the objectives in our

Table 1
Performance comparison before and after optimization for tetragonal core

Objectives		Load 1 (bending)	Load 2 (twisting)	Load 3 (torsion)
Maximum deflection (mm)	Original	0.331	0.165	0.412
	Optimized	0.225	0.101	0.273
Maximum stress (MPa)	Original	192.3	238.3	318.4
	Optimized	73.0	73.9	97.4
Fundamental frequency (Hz)	Original		24.17	
	Optimized		38.6	
Mass (kg)	Original		0.164	
	Optimized		0.100	

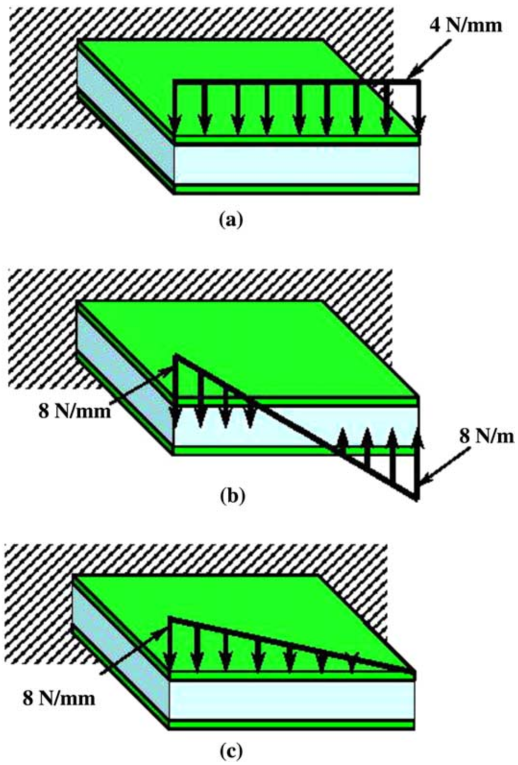

Fig. 5. Loading cases for tetragonal core panel: (a) bending, (b) twisting, and (c) torsion.

optimization model, here we keep the panel height as a design variable changeable throughout the optimization. From a practical point of view, the panel height may be fixed (or strut aspect ratio) in an optimization if so desired, as done by Wicks and Hutchinson (2001) and Deshpande and Fleck (2001), but the resulting minimum weight design would in general be greater than the global minimum.

Three loading cases are considered to study the performance of the tetrahedral truss-core panel, that is, bending, twisting and torsion loading conditions (see Fig. 5). The nodes at the clamped end of the structure act as boundary and are fully fixed. It is possible to incorporate into the optimization model more performances and loading cases without additional difficulty, if there are different design considerations and requirements.

In this optimization, maximum displacement, maximum Mises stress everywhere in the material, fundamental frequency and mass are set as objectives. The fundamental frequency is required to be increased, say, by at least one third to be equal or greater than 32 Hz (the frequency of the original structure is 24 Hz), and all other performances at three loading cases need to be minimized.

The simultaneous optimization under three loading cases can be solved quickly because the evaluation for performances can be obtained directly from FE analyses. The results for both the initial panel design and optimized design are listed in Table 1. From this table it can be observed that after the optimization iteration, a truss-core panel which is much stronger, stiffer, and lighter than the preliminary design is obtained. The modification of the design through the optimization can be seen in Table 2.

A more detailed description of the optimization process, parameter variation and optimization results will be discussed next by considering a more general truss-core panel example.

Table 2
Design variables for tetragonal core

Design variables	Original design	Optimized design
Panel thickness (mm)	10	17.18
Facesheet thickness (mm)	1.0	0.389
Cross-sectional area of strut (mm^2)	0.25	0.757

5.2. A more complicated scenario: optimization of a truss-core panel with plagihedral pyramidal cells under seven loading cases

5.2.1. Characteristics and design optimization problem

This is a new panel. A more generic design problem setting is considered in this optimization. The characteristics of the panel to be optimized are different from the tetrahedral unit cell depicted in Section 5.1. The details of a cell in the panel can be seen in Fig. 3d, where the number in a circle indicates the group to which the bar belongs. Since a group of struts are constructed perpendicular to the facesheets, its unit cell will be termed as the plagihedral pyramidal cell.

The same steel material is used for both the truss core and the solid facesheets with Young's modulus $E_s = 206$ GPa, Poisson's ratio $v = 0.3$, and mass density $\rho = 7850$ kg/m^3. The initial design had a fundamental frequency of 26.9 Hz with an overall mass of 0.176 kg (see Table 3). For this design, the frequency is required to be increased above 30 Hz, whilst other performances (i.e. structural mass, maximum displacement, and maximum stresses in both struts and facesheets) at all loading cases need to be minimized.

The design variables used for the optimization of a clamped truss-core panel (Fig. 3c) are

- distance D between neighboring vertices (Fig. 3d);
- panel height;
- facesheet thickness;
- cross-sectional areas of three group of struts, i.e.
  - vertical bars (group 1);
  - bars parallel to *xoz* or *yoz* planes (group 2);
  - bars diagnostically crossing the cells (group 3).

Table 3
Performance comparison before and after optimization for plagihedral pyramidal core

Objectives		Load 1	Load 2	Load 3	Load 4	Load 5	Load 6	Load 7
Maximum deflection	Original	0.176	0.129	0.419	0.369	0.349	0.352	0.096
(mm)	Optimized	0.154	0.101	0.381	0.352	0.208	0.207	0.064
Maximum stress in struts	Original	347	347	421	480	890	696	698
(MPa)	Optimized	210	210	217	490	250	393	402
Maximum stress in plates	Original	48	168	54	376	97	864	312
(MPa)	Optimized	18	17	577	546	561	564	125
Fundamental frequency	Original				26.9			
(Hz)	Optimized				30.3			
Mass (kg)	Original				0.176			
	Optimized				0.140			

In addition to panel height, sheet thickness and struts' cross-sectional areas, the distance between neighboring vertices (value *D* in Fig. 3d) are also taken as design variable in this optimization. Since the length and width of the total panel must be kept within 100 ± 10 mm during the optimization, the number of rows and columns of the cells will change accordingly with the change of the distance between neighboring vertices. The nodes at the clamped end of the structure act as boundary and their degrees of freedom are fully fixed, whilst the node coordinates are changeable as the panel height is varied.

Seven loading cases are included to reflect the performance of the truss-core panel in the optimization (Fig. 6):

Case 1: panel bending by two concentrated nodal forces (100 N each) at two corners;
Case 2: panel twisting by two concentrated nodal forces (100 N each) at two corners;
Case 3: panel planar bending by two concentrated nodal forces (5000 N each) at one corner;
Case 4: panel planar compression and twisting by two concentrated nodal forces (5000 N each) at two corners;
Case 5: panel planar stretching by four concentrated nodal forces (5000 N each) at two corners;
Case 6: panel shearing by four concentrated nodal forces (5000 N each) at two corners;
Case 7: panel bending by four bending moments (1000 N mm each) at two corners.

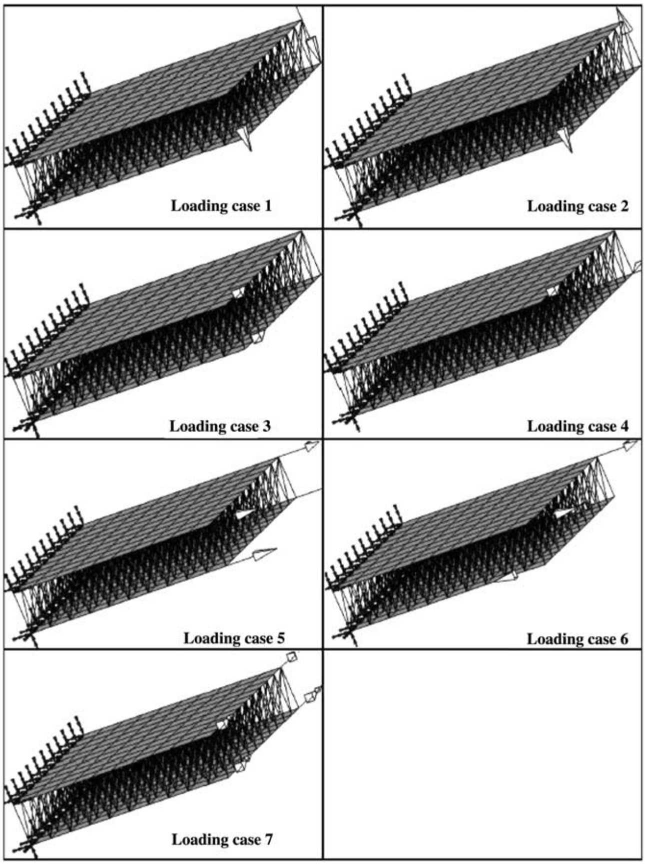

Fig. 6. Loading cases for plagihedral pyramidal core panel.

In this optimization, all the objective functions used in the Section 5.1 are included. Instead of minimizing the stresses considering the structure as a whole (including struts and facesheets), the stresses in struts and facesheets are separately listed and minimized.

The acceptable limits are considered in the optimization. The mass, maximum displacement, maximum stress in struts and maximum stress in facesheets at each loading case has each an upper acceptable limit, while the frequency has lower acceptable limit. In deriving the OPI, the performances and loading cases are weighted according to their importance.

5.2.2. Optimization results

By performing the optimization, the OPI is greatly enhanced, increasing from the original score of 10.61 to the optimized score of 83.59. The results shown in Figs. 7–10 demonstrate that all the performances have been successfully improved for all the loading cases. These figures demonstrate the convergence history from the preliminary design to a much better design based on the optimization criteria. From these figures, a desired steady convergence can be observed.

The performances of the system at all the loading cases for both the original design and the optimized system are presented in Table 3. The significant improvement in the design, following the optimization, can be observed as the following:

- largest structural displacement, 0.419 mm (at load case 3), is reduced to 0.381 mm (at load case 3);
- highest stress in the struts, 890 MPa (at load case 5), is reduced to 490 MPa (at load case 4);
- highest stress in the facesheets, 864 MPa (at load case 6), is reduced to 577 MPa (at load case 3);
- structural natural frequency is increased from the initial values of 26.9 Hz to the given best level value of 30.3 Hz;
- more significantly, whilst all of those mentioned above are achieved, the structural mass of 0.176 kg is decreased to 0.140 kg (a 20% reduction).

In the optimized structure, the highest strut stress 490 MPa and the highest facesheet stress 577 MPa are the stresses achieved under the optimization setting. Many steels may well be stronger than these. Note that the maximum stresses developed in some struts under certain loading cases exceed the yielding stress of steel, but, as previous discussed, no failure criterion is considered in the present study for simplicity. More

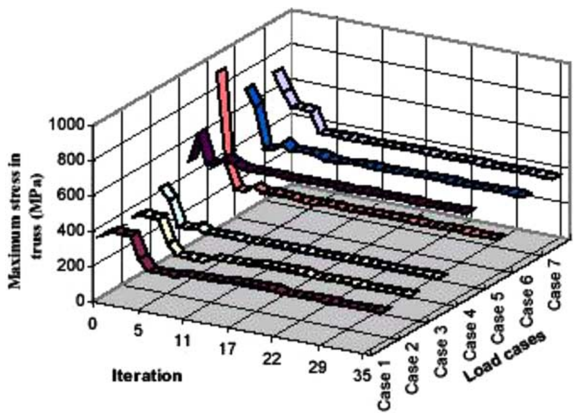

Fig. 7. Convergence history of maximum stress in the struts at different loading cases for plagihedral pyramidal core.

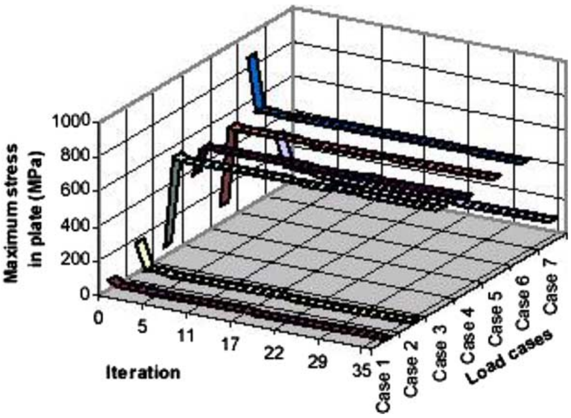

Fig. 8. Convergence history of maximum stress in the plates at different loading cases for plagihedral pyramidal core.

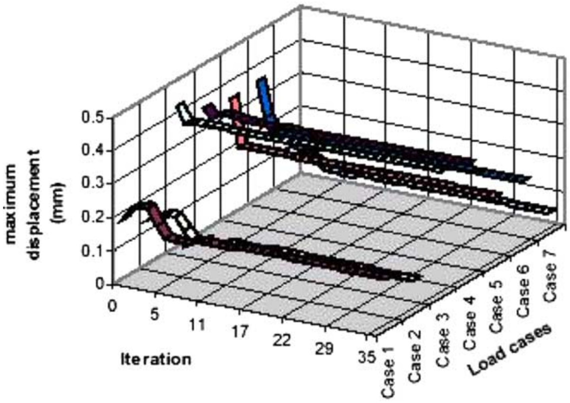

Fig. 9. Convergence history of maximum displacement at different loading cases for plagihedral pyramidal core.

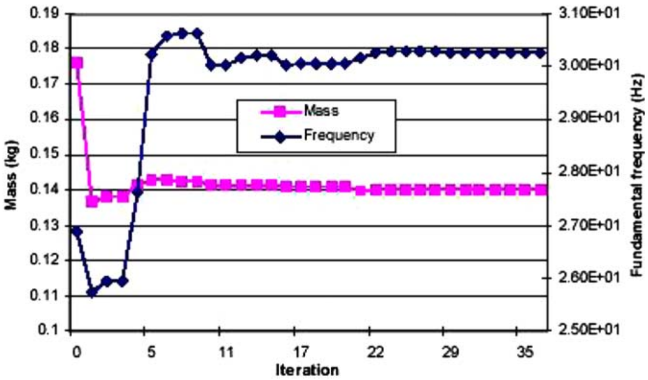

Fig. 10. Convergence history of structural mass and fundamental frequencies for plagihedral pyramidal core.

detailed optimization studies incorporating various failure modes (yielding, buckling, core shear etc.) will be carried out in the future.

Figs. 11–14 provide a more complete comparison between the characteristics of the truss-core panel before and after optimization at all the loading cases considered. The best level values and the given acceptable limits on each performance parameter at each loading case are also shown in these figures.

The mean values, SDs, PPIs and CPIs for the original and optimized systems are listed in Tables 4 and 5. It can be seen that the performance parameters of the optimized structure, at all loading cases, have much higher mean values and PPIs and much lower standard deviations than those in the original design. Also, at each loading case the material will behave in such a way that all the performances have increased reliability and decreased possibility of the system performing unsatisfactorily.

The optimized topology of the panel is shown in Fig. 15, which is to be compared with the original design of Fig. 3c. The optimized structure is stiffer and hence is expected to have higher modal frequencies than the original design. The modal analysis for the optimized design shows that the lowest natural frequency is increased to 30.3 Hz from its original value of 26.9 Hz, meeting the target value of 30 Hz. The variations of the design variables before and after the optimization are listed in Table 6. It can be seen from this table that the panel height is increased from 10 to 14.03 mm. Although this means that the struts would become longer (hence heavier), the total mass is still significantly reduced. This is due to the facts that reduction of facesheet thickness and reallocation of strut cross-sectional areas can offset the additional core weight.

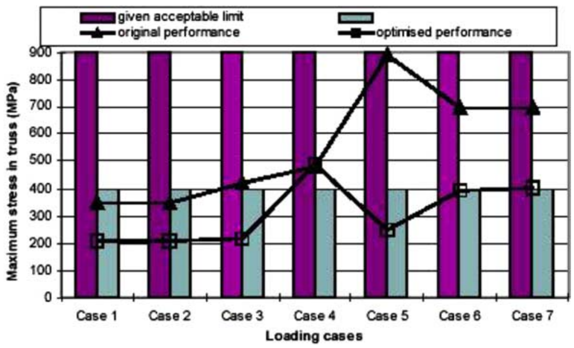

Fig. 11. Comparison of maximum stresses in struts between original and optimized designs for plagihedral pyramidal core.

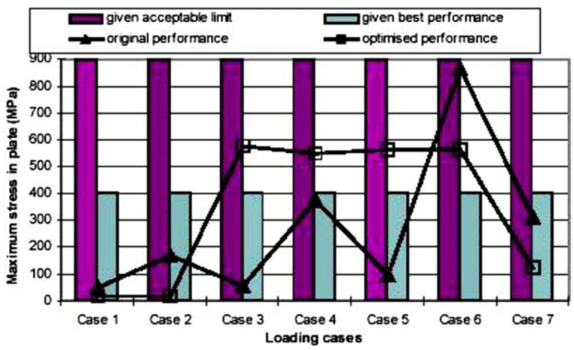

Fig. 12. Comparison of maximum stresses in plates between original and optimized designs for plagihedral pyramidal core.

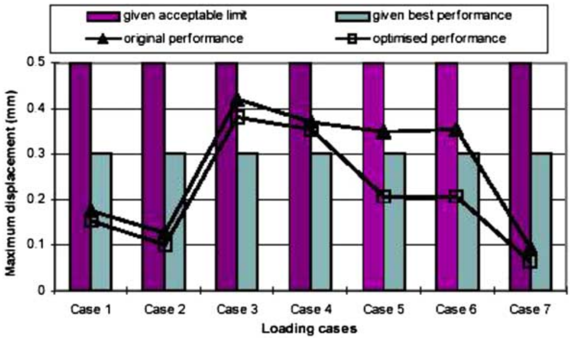

Fig. 13. Comparison of maximum displacements between original and optimized designs for plagihedral pyramidal core.

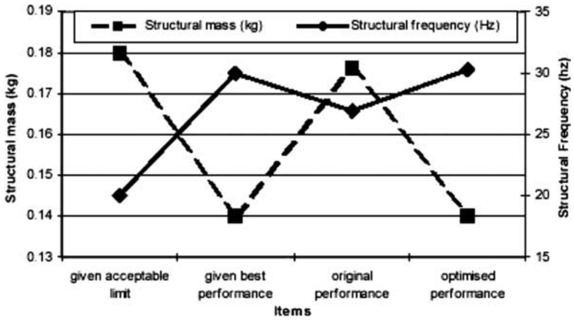

Fig. 14. Comparison of masses and frequencies between original and optimized designs for plagihedral pyramidal core.

Table 4
System parameter profile analysis of original and optimized designs for plagihedral pyramidal core

	Mean		Standard deviation		PPI	
	Original	Optimized	Original	Optimized	Original	Optimized
Maximum deflection (mm)	7.93	9.05	2.09	1.55	7.22	8.71
Maximum stress in struts (MPa)	6.62	9.74	3.57	0.63	1.14	9.69
Maximum stress in plates (MPa)	8.67	8.15	3.25	1.61	3.52	7.84
Fundamental frequency (Hz)	6.88	10.00	0.00	0.00	6.88	10.00
Mass (kg)	0.95	10.00	0.00	0.00	0.95	10.00

Table 5
Loading case profile analysis of original and optimized designs for plagihedral pyramidal core

	Mean		Standard deviation		CPI	
	Original	Optimized	Original	Optimized	Original	Optimized
Loading case 1	7.57	10.00	3.52	0.00	3.34	10.00
Loading case 2	7.57	10.00	3.52	0.00	3.34	10.00
Loading case 3	6.29	8.48	3.42	1.86	3.03	8.03
Loading case 4	6.56	8.53	3.06	1.25	3.19	8.35
Loading case 5	5.11	9.36	3.86	1.29	0.75	9.13
Loading case 6	4.00	9.34	2.82	1.31	1.69	9.11
Loading case 7	6.37	9.99	3.50	0.02	3.04	9.99

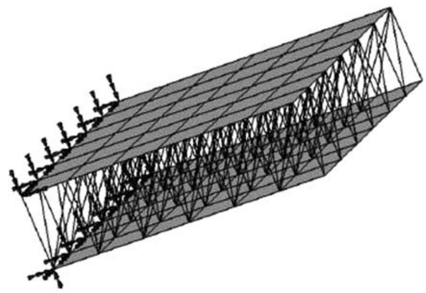

Fig. 15. Optimized plagihedral pyramidal core panel.

Table 6
Design variables for plagihedral pyramidal core

Design variables	Original design	Optimized design
Gap size D in x and y directions (mm)	10.0	18.2
Panel thickness (mm)	10.0	14.03
Facesheet thickness (mm)	1.0	0.991
Cross-sectional area of first group struts (mm^2)	0.25	0.454
Cross-sectional area of second group struts (mm^2)	0.25	0.350
Cross-sectional area of third group struts (mm^2)	0.25	0.300

6. Conclusions

In application, design specifications may impose specific constraints for different loading cases. For a structure with a single loading case, the optimum design for a certain objective function with various constraints can readily be found. However, the resulting optimum design may not satisfy the constraints for the structure in other loading cases and, more often than not, may even worsen other design objectives. These make the optimization often impractical.

The optimization technique presented here is shown to have an intrinsic applicability to multi-objective and multi-loading optimization. A systematic method is proposed to evaluate quantitatively the design of metallic truss-core panels. The evaluation indices of performances and loading cases are formulated and an overall performance index is presented to provide a scientific quantitative evaluation for the system. The optimization addresses various parameters, their interrelations, and various loading cases. The method is of great flexibility to design lightweight materials with maximum satisfaction. Like any non-linear programming method, the method presented here may not necessarily make every performance at every loading case reach its specified aspiration value (unless extremely lucky), especially when conflicting objectives and many loading cases are simultaneously considered in an optimization. This is due to the Pareto feature of some optimization problems, and there are tradeoffs when reaching a globally acceptable solution. However, the proposed methodology makes it feasible to optimize complicated structures and systems for different loading scenarios. One of its primary benefits in the application is the elimination of the potentially expensive separate optimizations for each objective. For solutions of structural problems, the optimization method is coupled with commercial finite element codes to determine system responses as functions of design variables.

The new development of metallic truss-core sandwich panels offers structural designers a wide range of new degrees of freedom in terms of simultaneous optimization of structural configuration and structural material. The application of the multi-objective optimization method on these panels appears very encouraging. However, the focus of this study has been placed on the development of a new optimization method for multi-objective and multi-loading cases, with lightweight truss-core panels used as the demonstrating example. Although two different types of core are considered, the loading cases considered for the tetragonal core are different from those associated with the plagihedral pyramidal core, as the intention was not to compare their performance specifically. It will be the future work that the performances (including mass) of different sandwich panels with various truss core systems or honeycomb cores are compared from the stiffness, strength, weight, manufacturing, and cost points of view. These may be done by optimizing the panels under identical optimization setting using the techniques described here.

Acknowledgements

This work was supported partly by the US Office of Naval Research (grant number N000140110271), and partly by the UK Engineering and Physical Sciences Research Council (EPSRC grant number EJA/U83).

References

Chiras, S., Mumm, D.R., Evans, A.G., Wicks, N., Hutchinson, J.W., Dharmasena, K., Wadley, H.N.G., Fichter, S., 2002. The structural performance of near-optimized truss core panels. Int. J. Solids Struct. 39, 4093–4115.

Deshpande, V.S., Fleck, N.A., 2001. Collapse of truss-core sandwich beams in 3-point bending. Int. J. Solids Struct. 38, 6275–6305.

Deshpande, V.S., Fleck, N.A., Ashby, M.F., 2001. Effective properties of the octet-trucc lattice material. J. Mech. Phys. Solids 49, 1747–1769.

Evans, A.G., Hutchinson, J.W., Fleck, N.A., Ashby, M.F., Wadley, H.N.G., 2001. The topological design of multifunctional cellular metals. Prog. Mater. Sci. 46, 309–327.

Hyun, S., Karlsson, A.M., Torquato, S., Evans, A.G., 2002. Simulated properties of kagome and tetragonal truss core panels. Technical Report, Princeton Materials Institute, Princeton University, 2002.

Kim, T., Zhao, C.Y., Lu, T.J., Hodson, H.P., in press. Convective heat dissipation with lattice-frame materials. Mech. Mater.

Liu, J.-S., Hollaway, L., 1998. Integrated structure-electromagnetic optimization of large reflector antenna systems. Int. J. Struct. Optimiz. 16, 29–36.

Liu, J.-S., Hollaway, L., 2000. Design optimization of composite panel structures with stiffening ribs under multiple loading cases. Comput. Struct. 78, 637–647.

Liu, J.-S., Thompson, G., 1996. The multi-factor design evaluation of antenna structures by parameter profile analysis. In: Proceedings of the Institution of Mechanical EngineersJ. Eng. Manuf. 210 B5, 449–456.

Lu, T.J., Evans, A.G., 2002. Design of a high authority flexural actuator using an electrostrictive polymer. Sensor Actuator A 99, 290–296.

Lu, T.J., Evans, A.G., Hutchinson, J.W., 2001. Optimal design of a flexural actuator. J. Mech. Phys. Solids 49, 2071–2093.

Sypeck, D.J., Wadley, H.N.G., 2001. Multifunctional microtruss laminates: textile synthesis and properties. J. Mater. Res. 16, 890–897.

Thompson, G., Goeminne, J., 1993. The design review of engineering systems including maintainability and reliability using parameter profiles. In: Int. Conf. on Engineering Design, The Hague, August 1993.

Tian, J., Kim, T., Lu, T.J., Hodson, H.P., Sypeck, D.J., Wadley, H.N.G., submitted for publication. The effects of topology upon fluid-flow and heat-transfer within cellular copper structures. Int. J. Heat Mass Transfer.

Wallach, J.C., Gibson, L.G., 2001. Mechanical behavior of a three-dimensional truss material. Int. J. Solids Struct. 38, 7181–7196.

Wicks, N., Hutchinson, J.W., 2001. Optimal truss plates. Int. J. Solids Struct. 38, 5165–5183.